

Graphing Techniques: Transformations

Parent Graph: $y = f(x)$

Offspring: Transformations of the parent graph.

	$f(x) = x^2$	$f(x) = \sqrt{x}$	$f(x) = \frac{1}{x}$	Effect on Parent Graph
$y = f(x) + 2$				
$y = f(x) - 2$				
$y = f(x + 2)$				
$y = f(x - 2)$				
$y = 2f(x)$				
$y = \frac{1}{2}f(x)$				
$y = f(2x)$				
$y = f\left(\frac{1}{2}x\right)$				
$y = -f(x)$				
$y = f(-x)$				

When graphing a transformed graph based on an equation, apply transformations in the following order:

- 1.
- 2.
- 3.

Examples: List the transformations in the appropriate order:

Parent graph: $y = \sqrt{x}$

a) $y = -\frac{1}{2}\sqrt{x+3}$

b) $y = 5\sqrt{-4x+3}$

c) $y = 3\sqrt{-2x+9}$

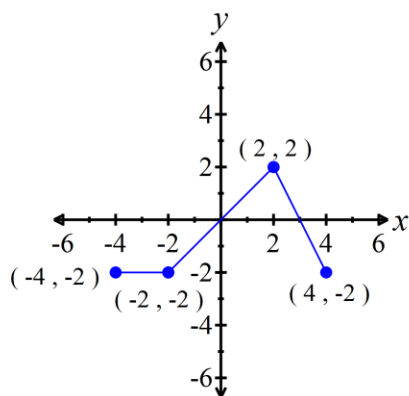
Parent graph: $f(x) = |x|$

a) $f(x) = 4|x-2|+7$

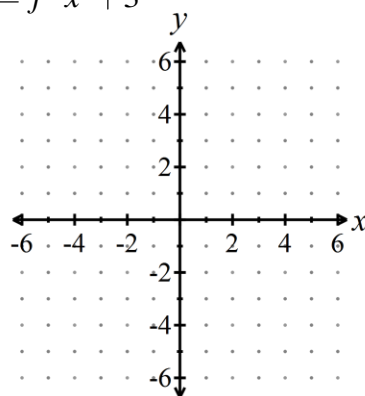
b) $f(x) = -|x+5|-3$

c) $f(x) = -\left|\frac{x}{3}+2\right|$

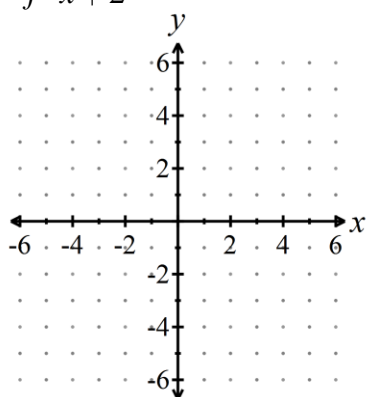
Example: The graph of a function f is illustrated below. Use the graph of f as the first step towards graphing each of the following functions:



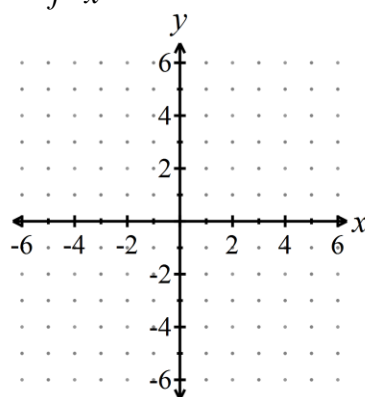
a) $F(x) = f(x) + 3$



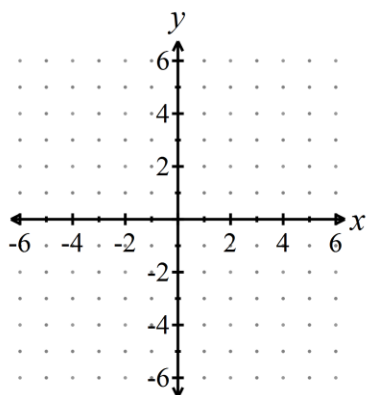
b) $G(x) = f(x) + 2$



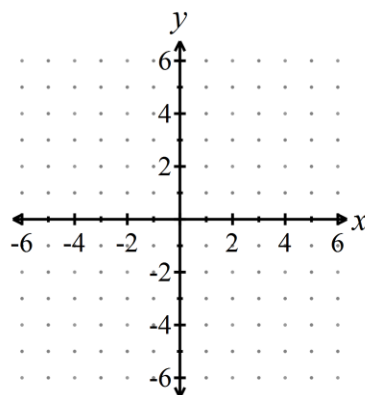
c) $P(x) = -f(x)$



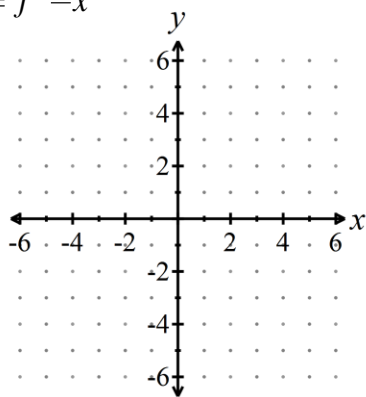
d) $H(x) = f(x+1) - 2$



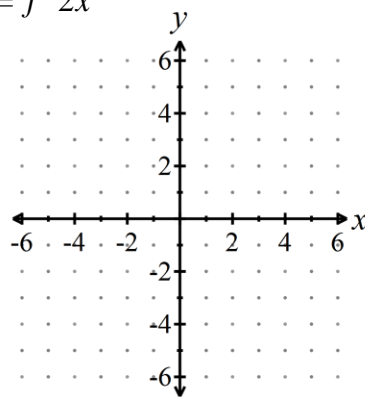
e) $Q(x) = 2f(x)$



f) $g(x) = f(-x)$

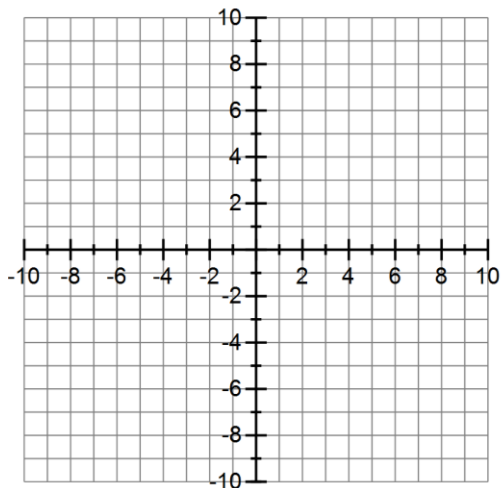


g) $h(x) = f(2x)$

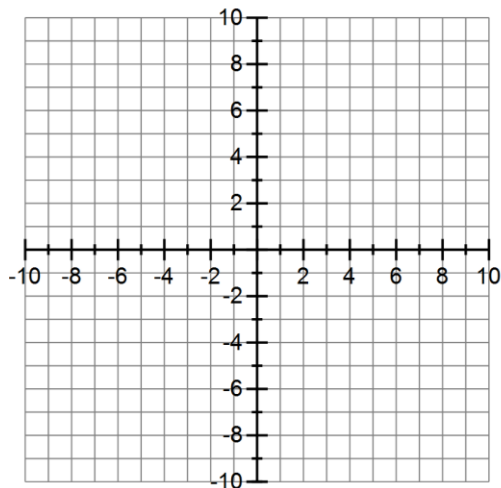


Examples: Graph the following:

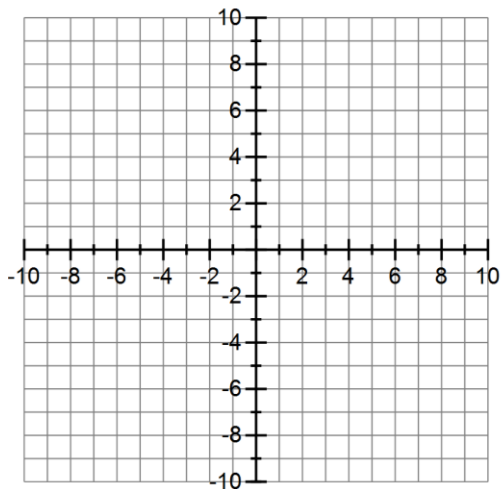
a) $f(x) = (x-1)^3 + 2$



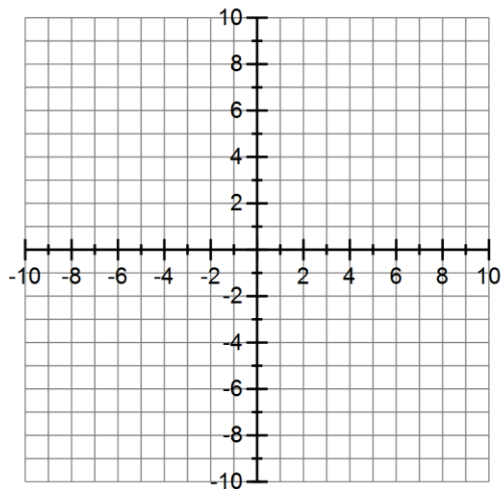
b) $g(x) = 2|x+1| - 3$



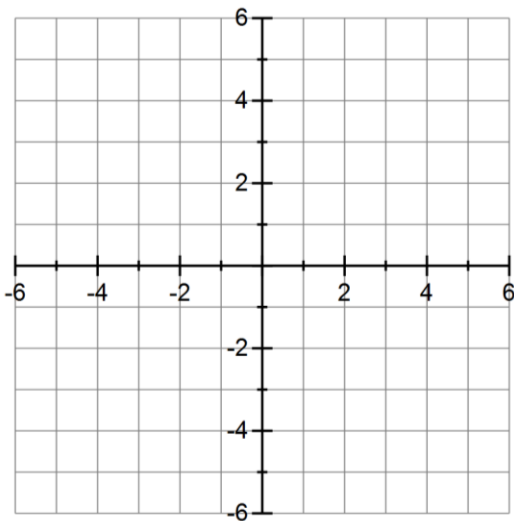
c) $f(x) = \sqrt{-(x-3)} + 2$



d) $g(x) = -\sqrt[3]{2x}$



e) $h(x) = \frac{3}{(x+2)}$



Example: Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $g(x) = x^3$.

1. Shift down 4 units
2. Reflect across y -axis
3. Vertical compression by a factor of $1/2$

Example: Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $h(x) = \sqrt{x}$.

1. Vertical stretch by a factor of 3
2. Move left 5 units
3. Reflect across the y -axis

Example: Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $f(x) = |x|$.

1. Horizontal compression by a factor of $1/2$
2. Move up 6 units
3. Reflect across the x -axis

Summary of Graphing Transformations:

To Graph:	Draw the Graph of $y = f(x)$ and:	Functional Change to $y = f(x)$:
Reflection About the x-axis $y = -f(x)$	Reflect the graph of f about the x -axis.	Multiply $f(x)$ by -1 .
Reflection About the y-axis $y = f(-x)$	Reflect the graph of f about the y -axis.	Replace x by $-x$.
Vertical Stretches & Compressions $y = af(x), a > 0$	Multiply each y -coordinate of $y = f(x)$ by a . This stretches the graph of f vertically if $a > 1$. This compresses the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a .
Horizontal Stretches & Compressions $y = f(bx), b > 0$	Divide each x -coordinate of $y = f(x)$ by b . This stretches the graph of f horizontally if $0 < b < 1$. This compresses the graph of f horizontally if $b > 1$.	Replace x by bx .
Vertical Shifts $y = f(x) + k, k > 0$ $y = f(x) - k, k > 0$	Raise the graph of f by k units. Lower the graph of f by k units.	Add k to $f(x)$. Subtract k from $f(x)$.
Horizontal Shifts $y = f(x - h), h > 0$ $y = f(x + h), h > 0$	Shift the graph of f to the right by h units. Shift the graph of f to the left by h units.	Replace x by $x - h$. Replace x by $x + h$.